On the complexity of Matsui's attack against DES

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Outline

Matsui's linear cryptanalysis against 16-rounds DES, as proposed at Crypto'94.

- Historical Overview
- Experimental Results
- Theoretical Analysis
- Conclusion

Linear Cryptanalysis Performances: Historical Overview

- [Matsui, Eurocrypt'93, Crypto'94] Linear cryptanalysis, first experimental implementation

- [Blöcher-Dichtl, FSE'94] Some observations on the application of the piling-up lemma

- [Nyberg, Eurocrypt'94] Linear hull concept

- [Harpes-Kramer-Massey, Eurocrypt'95] Generalization of linear cryptanalysis

- [Vaudenay, 1999] Statistical cryptanalysis concept

- [Kukorelly, 1999] Theoretical study on the piling-up lemma application

- [Selçuk, Indocrypt'00] Bias estimation in linear cryptanalysis

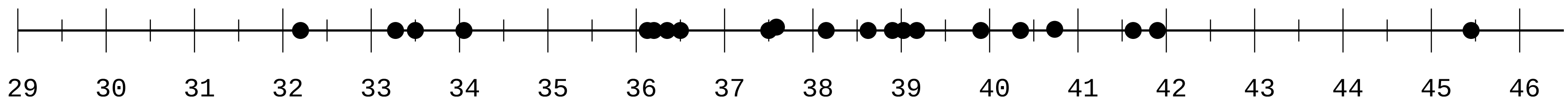
Experiment Description

- Matsui attack has been implemented using today's technology
- Fast DES routine (bitsliced implementation on the Intel MMX architecture)
- Idle time of 12 - 18 CPUs
- 3-7 days to produce and analyse 2^{43} known pairs
- The experiment has run 21 times

Experimental Results (1)

- Widely accepted attack complexity: *Given 2^{43} known pairs, it is possible to recover the key with a success probability of 85 % within $\mathcal{C}^{est}_{(0.85)} = 2^{43}$ DES computations.*

- Real complexity $\mathcal{C}_{(0.85)}$ seems to be lower (logarithmic scale):



- Experimental results suggest: *Given 2^{43} known pairs, it is possible to recover the key with a success probability of 85 % within $\mathcal{C}_{(0.85)} = 2^{41}$ DES computations.*

Experimental Results (2)

Other experimental results:

- Given 2^{43} known pairs, $\mathcal{C}_{(0.5)} \approx 2^{38.5}$.
- Given $2^{42.5}$ known pairs, $\mathcal{C}_{(0.5)} \approx 2^{42}$.
- Given 2^{40} known pairs, $\mathcal{C}_{(0.5)} \approx 2^{51.5}$.

Analysis (1)

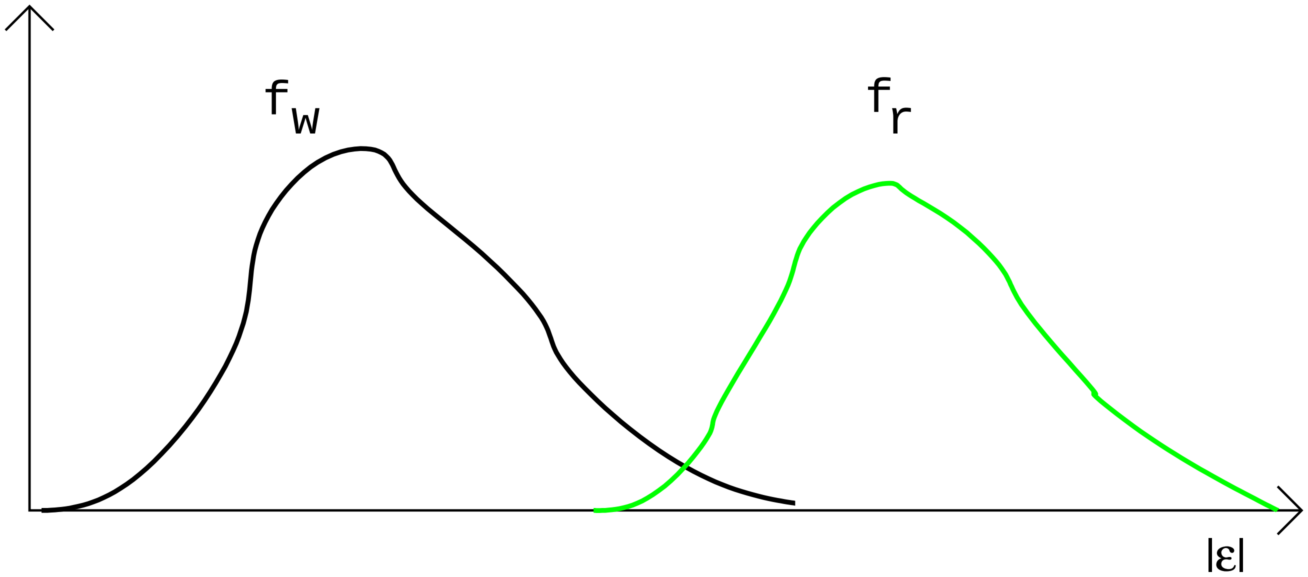
- Linear expression : $P_{[i_1,\ldots,i_r]} \oplus C_{[j_1,\ldots,j_s]} = K_{[k_1,\ldots,k_t]}$

- The expression must be biased in order to be useful: $\Pr[\text{Expression holds}] = \frac{1}{2} + \epsilon, |\epsilon| > 0$.

- Wrong-key randomization hypothesis:

$$\frac{\left|\Pr[\text{Expression holds}|\text{right key}] - \frac{1}{2}\right|}{\left|\Pr[\text{Expression holds}|\text{wrong key}] - \frac{1}{2}\right|} \gg 1$$

Analysis (2)



- Statistical Cryptanalysis Concept [Vaudenay, 1995]

- Counting / Analysis / Sorting / Searching phases

- Complexity

- Success Probability : key bits sum guessing, success within a given complexity

- Complexity is function of the right subkey rank Ψ in the candidates list

Analysis (3)

- *Assumption 1*: Bias produced by a wrong key is independant of the key

- *Assumption 2*: Bias produced by the right key is independant of the ones produced by wrong keys

- *Assumption 3*: The distribution of the biases is well approximated by a normal law

- $n - 1$ wrong candidates follow a probability density f_W, the right one follows f_R.

Theorem 1

$$\Pr\left[\Psi \leq \psi\right] = \int_{-\infty}^{+\infty} B_{n+1-\psi,\psi}(F_W(x)) f_R(x) dx$$

and

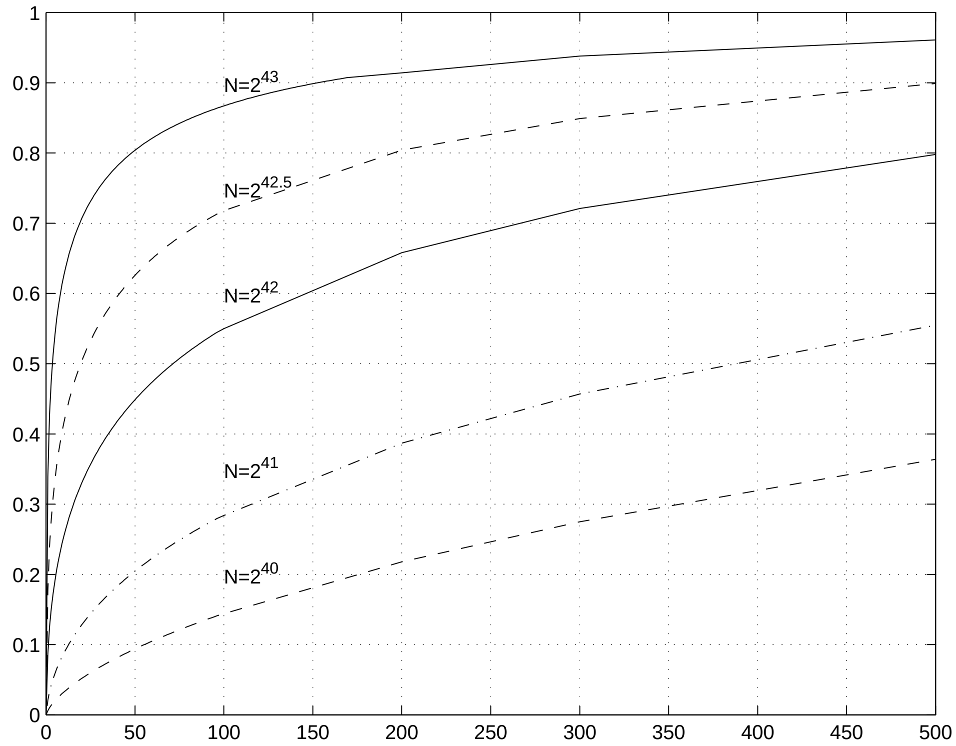
$$E\left[\Psi\right] = 1 + n\left(1 - \int_{-\infty}^{+\infty} f_R(x) F_W(x) dx\right)$$

where

$$B_{a,b}(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1}(1-t)^{b-1} dt$$

is the incomplete beta function of order (a,b)*.*

Analysis (4)



Theoretical rank distribution ($\epsilon_w = 0$ and $\epsilon_R =$ piling-up approximation) for various amount of known pairs.

Analysis (5)

Some observations:

- Wrong-key randomization hypothesis holds well

- $\widehat{\epsilon}_r - \epsilon_r$ is small

- $\widehat{\epsilon}_w \neq 0$, but it doesn't matter a lot

- The experimental variances are *a lot* smaller than the theoretical ones.

Conclusion

- Experimental complexity analysis
- Theoretical analysis
- Partial inacurracy of the model explained by experimental observations